ON NATURAL LEAPING CONVERGENTS OF REGULAR CONTINUED FRACTIONS AND AN APPLICATION TO LINEAR FRACTIONAL TRANSFORMATIONS

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(submitted paper)

Let $\sigma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ be the coefficient matrix with nonzero determinant $\Delta := \det \sigma$ of the linear fractional transformation $\sigma(x) = (ax+b)/(cx+d)$, where $a, b, c, d \in \mathbb{Z}$. In this paper we introduce a new concept called natural leaping convergents. Such convergents are defined in terms of combinatorial pairings between the rational convergents of an irrational number ξ and those of it's transformation $\sigma(\xi)$. The structural properties of natural leaping convergents are then studied and sufficient conditions are given for determining whether $\sigma(p_n/q_n) \in \mathcal{C}(\sigma(\xi)) \cap \sigma(\mathcal{C}(\xi))$, where $\mathcal{C}(\eta)$ denotes the set of convergents of the number η. We then present a theorem for expressing the convergents for all quadratic irrationals in closed form and establish for the quadratic irrationals, as well as for transcendental numbers, that given a suitable value for Δ, there are at most finitely many convergents p_n/q_n of ξ satisfying $\sigma(p_n/q_n) \in \mathcal{C}(\sigma(\xi)) \cap \sigma^*(\mathcal{C}(\xi))$, where * indicates the added property that $\gcd(ap_n + bq_n, cp_n + dq_n) = 1$.

AMS Subject Classification: Primary: 11A55, 11J70, 11B37 ; Secondary: 11J04, 11B83.
Keywords: Continued fractions, Recurrences, Linear Fractional Transformations, Rational Approximations.