

# DIOPHANTINE ANALYSIS AROUND $[1, 2, 3, \dots]$

Carsten Elsner and Christopher Robin Havens

(submitted paper)

The transcendental value  $\mathfrak{z}$  of the regular infinite continued fraction  $[1, 2, 3, 4, 5, \dots]$  is a ratio of the values of modified Bessel functions. In this paper our diophantine analysis around  $\mathfrak{z}$  takes its starting point with its rational convergents and deals with an asymptotic approximation formula for  $\mathfrak{z}$  and with the construction of a sequence of quadratically irrational approximations using these convergents. Finally, we study various error sums for  $\mathfrak{z}$  which are also defined by the rational convergents.

Dedicated to the memory of Professor Eduard Wirsing (1931 - 2022)

**AMS Subject Classification:** Primary: 11A55; Secondary: 11J70, 33C10.

**Keywords:** Continued fractions, error sums, recurrences, Bessel functions.