A CRITERION OF ALGEBRAIC INDEPENDENCE OF VALUES OF MODULAR FUNCTIONS AND AN APPLICATION TO INFINITE PRODUCTS INVOLVING FIBONACCI AND LUCAS NUMBERS

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The aim of this paper is to give a criterion of algebraic independence for the values at the same point of two modular functions under certain conditions. As an application, we show that any two infinite products in

$$\prod_{n=1}^{\infty}\left(1+\frac{1}{F_n}\right), \quad \prod_{n=3}^{\infty}\left(1-\frac{1}{F_n}\right), \quad \prod_{n=1}^{\infty}\left(1+\frac{1}{L_n}\right), \quad \prod_{n=2}^{\infty}\left(1-\frac{1}{L_n}\right)$$

are algebraically independent over $\mathbb{Q}$, where $\{F_n\}$ and $\{L_n\}$ are the Fibonacci and Lucas sequences, respectively. The proof of our main theorem is based on the properties of the field of all modular functions for the principal congruence subgroup, together with a deep result of Yu. V. Nesterenko on algebraic independence of the values of the Eisenstein series.

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