ON LANGUAGES FORMED BY SETS OF INTEGERS - A BRIDGE BETWEEN AUTOMATA THEORY AND NUMBER THEORY

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(submitted paper)

In this paper we briefly introduce the concepts of deterministic and nondeterministic finite automata and of regular languages. Based on the Myhill-Nerode theorem and on the pumping lemma for regular languages, we prove the irregularity of languages formed by sets of primes, Fibonacci numbers, Fermat numbers and other sets of integers, which are considered as languages built from the alphabet $\{0, 1, \ldots, 9\}$ of their digits. For this goal, different languages require different number theoretic tools. Analytic and algebraic arguments are used, e.g., Kronecker's approximation theorem, Bezout's theorem on linear diophantine equations in two unknowns, and the theory of residue class groups and their subgroups.
The results can be generalized to representations of integers in different digit systems.

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