

ON LINEAR INDEPENDENCE RESULTS FOR VALUES OF JACOBI THETA-CONSTANTS

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In 2019 Elsner, Luca and Tachiya proved that the values of the Jacobi-theta constants $\theta_3(m\tau)$ and $\theta_3(n\tau)$ are algebraically independent over \mathbb{Q} for distinct integers m, n under some conditions on τ . On the other hand, in 2018 Elsner and Tachiya also proved that three values $\theta_3(m\tau), \theta_3(n\tau)$ and $\theta_3(\ell\tau)$ are algebraically dependent over \mathbb{Q} . In this article we prove the non-vanishing of linear forms in $\theta_3(m\tau), \theta_3(n\tau)$ and $\theta_3(\ell\tau)$ under various conditions on m, n, ℓ , and τ . Among other things we prove that for odd and distinct positive integers $m, n > 3$ the three numbers $\theta_3(\tau), \theta_3(m\tau)$ and $\theta_3(n\tau)$ are algebraically independent over $\overline{\mathbb{Q}}$ when τ is an algebraic number of some degree greater or equal to 3. In some sense this fills the gap between the above-mentioned former results on theta constants. A theorem on the linear independence over $\mathbb{C}(\tau)$ of the functions $\theta_3(a_1\tau), \dots, \theta_3(a_m\tau)$ for distinct positive real numbers a_1, \dots, a_m is also established.

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