

Algebraic independence results for the values of the theta-constants and some identities

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In the present work, we give algebraic independence results for the values of the classical theta-constants $\vartheta_2(\tau)$, $\vartheta_3(\tau)$, and $\vartheta_4(\tau)$. For example, the two values $\vartheta_\alpha(m\tau)$ and $\vartheta_\beta(n\tau)$ are algebraically independent over \mathbb{Q} for any τ in the upper half-plane when $e^{\pi i\tau}$ is an algebraic number, where $m, n \geq 1$ are integers and $\alpha, \beta \in \{2, 3, 4\}$ with $(m, \alpha) \neq (n, \beta)$. This algebraic independence result provides new examples of transcendental numbers through some identities found by S. Ramanujan. We additionally give some explicit identities among the three theta-constants in particular cases.

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