

# Algebraic independence results for the values of the theta-constants and some identities

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In the present work, we give algebraic independence results for the values of the classical theta-constants  $\vartheta_2(\tau)$ ,  $\vartheta_3(\tau)$ , and  $\vartheta_4(\tau)$ . For example, the two values  $\vartheta_\alpha(m\tau)$  and  $\vartheta_\beta(n\tau)$  are algebraically independent over  $\mathbb{Q}$  for any  $\tau$  in the upper half-plane when  $e^{\pi i\tau}$  is an algebraic number, where  $m, n \geq 1$  are integers and  $\alpha, \beta \in \{2, 3, 4\}$  with  $(m, \alpha) \neq (n, \beta)$ . This algebraic independence result provides new examples of transcendental numbers through some identities found by S. Ramanujan. We additionally give some explicit identities among the three theta-constants in particular cases.

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