IRRATIONALITY MEASURES OF NUMBERS WITH CONVERGENTS IN ARITHMETIC PROGRESSIONS

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(submitted paper)

In this paper we combine two aspects of rational approximations to real numbers: on the one side the irrationality measure, on the other side the restrictions of numerators and denominators of the approximating fractions to arithmetic progressions. First, we show that real numbers with a prescribed irrationality measure and with infinitely many convergents satisfying the arithmetic restrictions produce a dense subset on the real line. Then we introduce two modified irrationality exponents of real numbers which measure the approximability of numbers by fractions with arithmetic restrictions. We give examples for the relationsships between these two irrationality measures. A special focus lies on the mass-theoretical largest set of numbers with irrationality measure two. We generalize a former result of the author (1999) concerning best-possible constants in a theorem of S.Uchiyama (1980). Finally, we prove for numbers $e^{1/t}$ with $t = 1, 2, \ldots$ and for the limit of a Rogers-Ramanujan continued fraction a wide range of arithmetical restrictions on their convergents, under which these numbers can be better approximated than it is known in the general case by Uchiyama's theorem.

2010 MS Classification numbers: 11J70, 11J82
Key Words: Continued fractions, irrationality exponents