

ALGEBRAIC RESULTS FOR THE VALUES $\vartheta_3(m\tau)$ AND $\vartheta_3(n\tau)$ OF THE JACOBI THETA-CONSTANT

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Let $\vartheta_3(\tau) = 1 + 2 \sum_{\nu=1}^{\infty} e^{\pi i \nu^2 \tau}$ denote the classical Jacobi theta-constant. In this paper, we prove that the two values $\vartheta_3(m\tau)$ and $\vartheta_3(n\tau)$ are algebraically independent over \mathbb{Q} for any τ such that $q = e^{\pi i \tau}$ is an algebraic number, where $m, n \geq 2$ are distinct integers.

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