ON ERROR SUMS FORMED BY RATIONAL APPROXIMATIONS WITH SPLIT DENOMINATORS

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In this paper we consider error sums of the form

$$\sum_{m=0}^{\infty} \varepsilon_m \Big(b_m \alpha - \frac{a_m}{c_m} \Big),$$

where α is a real number, a_m, b_m, c_m are integers, and $\varepsilon_m = 1$ or $\varepsilon_m = (-1)^m$. In particular, we investigate such sums for

$$\alpha \in \big\{ \pi, e, e^{1/2}, e^{1/3}, \ldots, \log(1+t), \zeta(2), \zeta(3) \big\}$$

and exhibit some connections between rational coefficients occurring in error sums for Apéry's continued fraction for $\zeta(2)$ and well-known integer sequences. The concept of the paper generalizes the theory of ordinary error sums, which are given by $b_m = q_m$ and $a_m/c_m = p_m$ with the convergents p_m/q_m from the continued fraction expansion of α.

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