

# ON EXPONENTIAL-TYPE SUMS FORMED BY SOLUTIONS OF PELL'S EQUATION

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The object of this paper are the exponential generating functions of the non-negative integer solutions  $X_n$  and  $Y_n$  of Pell's equations  $X^2 - DY^2 = \pm 1$ . When  $(X_n, Y_n)$  run through all solutions of a fixed Pell equation, we prove the algebraic independence over  $\mathbb{Q}$  of two associated exponential functions at the same non-vanishing algebraic point. We also investigate the algebraic independence of exponential generating functions at non-vanishing algebraic points associated with solutions  $(X_n, Y_n)$  of distinct Pell equations  $X^2 - D_i Y^2 = 1$  ( $i = 1, \dots, n$ ).

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