ON A GENERALIZATION OF HALL'S THEOREM FOR SUMS OF CONTINUED FRACTIONS

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Let $1 \leq M < N$ be integers, and denote by $CF(M,N)$ the set of all irrationals from $[0,1]$ whose partial quotients a_ν of the continued fraction expansion satisfy $M \leq a_\nu \leq N \ \ (\nu \geq 1)$. It is proved that every real number can be expressed as a sum of an integer and m irrationals from $CF(M,N)$, where a lower bound for m is explicitly given. This includes Hall's theorem, and, e.g., the case where $M = N-1\,,\ m = N^2+1$.
Although the main theorem is a special case of a result of S. Astels from 2000, our proof is focused on a generalization of a lemma of Hall concerning the iterated thinning of intervals. It is easier to handle this tool than the theory of Cantor sets underlying Astels' approach from 2000.

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