ERRORSUMS FOR THE VALUES OF THE EXPONENTIAL FUNCTION

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Let p_n/q_n ($n \geq 0$) denote the n-th convergent from the continued fraction expansion of some real number α. There are various studies about series formed by their error terms $q_n\alpha - p_n$. In this article we investigate their generating function

$$\mathcal{E}(\alpha, t) := \sum_{n\geq 0} t^n |q_n\alpha - p_n|,$$

under various analytical and arithmetical aspects, and focus on values of the Exponential Function. Furthermore we introduce a new kind of error sum function

$$\mathcal{E}_{NB}(\alpha, t) := |\alpha - a_0| + \sum_{\nu=1}^{\infty} t^{\nu} \sum_{1\leq b\leq a_\nu} \big|(bq_{\nu-1} + q_{\nu-2})\alpha - (bp_{\nu-1} + p_{\nu-2})\big|,$$

which takes all the minor convergents of α into account.

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