ALGEBRAIC INDEPENDENCE OF VALUES OF EXPONENTIAL TYPE POWER SERIES

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In this paper we study variants of exponential type power series of the form $f_k(x) = \sum_{n=0}^{\infty} c_{k,n}\alpha^n/n!$ $(k = 1,\ldots,q)$ with real or complex coefficients $c_{k,n}$ and with a nonzero algebraic number α. We find all subsets of $\{f_1(\alpha),\ldots,f_q(\alpha)\}$ which are algebraically independent over $\mathbb{Q}$. We apply our method to series with periodic sequences, i.e., with $c_{k,n} = 1$ if $n \equiv k \pmod{q}$, $c_{k,n} = 0$ otherwise and also with $c_{k,n} = \{(an+k)/q\}$ for coprime integers a and q, where $\{x\}$ is the fractional part of a real number x. More applications deal with various series formed by Fibonacci and Lucas numbers, e.g., $c_{k,n} = F_{an+k}$. All the algebraic independence results are finally reduced to the Lindemann-Weierstrass theorem.