TRANSCENDENCE RESULTS AND CONTINUED FRACTION EXPANSIONS OBTAINED FROM A COMBINATORIAL SERIES

Carsten Elsner and Arne Klauke

(Journal of Combinatorics and Number Theory 5, no. 1 (2013), 53-79)

In this paper we study various properties of the combinatorial series

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n+1)(2n+3)\cdots(2n+2k+1)\binom{2n}{n}}\,.$$

Our investigations involve trinomial coefficients, the Catalan numbers, and the Fibonacci numbers. Starting from this series, we show the transcendence of several numbers by Baker's method on linear forms in logarithms. Using a linear transformation of the above series, we produce continued fraction expansions for many values including $1/\pi$, $\sqrt{3}/\pi$, $\sqrt{5}/\log\rho$, where ρ is the Golden Number. We obtain irrationality exponents for some of these numbers. Moreover, we construct a new sequence of transcendental numbers, whose irrationality measures converge to 2.

MR 2010 Subject Classification:
Primary 11A55; Secondary 11B75, 11B39, 05A10, 11J04, 11J82, 11J81.

Key words: Combinatorial series, Catalan numbers, Fibonacci numbers, transcendence, Zeilberger algorithm, continued fractions, irrationality exponents.