

ON ALGEBRAIC RELATIONS FOR RAMANUJAN'S FUNCTIONS

Carsten Elsner and Iekata Shiokawa

Ramanujan Journal 29 (2012), 273–294

DOI: 10.1007/s11139-012-9384-8

Let P, Q , and R denote the Ramanujan Eisenstein series. We compute algebraic relations in terms of $P(q^i)$ ($i = 1, 2, 3, 4$), $Q(q^i)$ ($i = 1, 2, 3$), and $R(q^i)$ ($i = 1, 2, 3$). For complex algebraic numbers q with $0 < |q| < 1$ we prove the algebraic independence over \mathbb{Q} of any three-element subset of $\{P(q), P(q^2), P(q^3), P(q^4)\}$ and of any two-element subset of $\{Q(q), Q(q^2), Q(q^3)\}$ and $\{R(q), R(q^2), R(q^3)\}$, respectively. For all the results we use some expressions of $P(q^{i_1}), Q(q^{i_2})$, and $R(q^{i_3})$ in terms of theta constants. Computer-assisted computations of functional determinants and resultants are essential parts of our proofs.

MR 2010 Subject Classification: 11J85, 11J91, 11F27.

Key words: Algebraic independence, Ramanujan's functions, Nesterenko's theorem, Independence criterion, Resultants