

ON ERROR SUM FUNCTIONS FORMED BY CONVERGENTS OF REAL NUMBERS

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Let p_m/q_m denote the m -th convergent ($m \geq 0$) from the continued fraction expansion of some real number α . We continue our work on error sum functions defined by

$$\mathcal{E}(\alpha) := \sum_{m \geq 0} |q_m \alpha - p_m| \quad \text{and} \quad \mathcal{E}^*(\alpha) := \sum_{m \geq 0} (q_m \alpha - p_m)$$

by proving a new density result for the values of \mathcal{E} and \mathcal{E}^* . Moreover, we study the function \mathcal{E} with respect to continuity and compute the integral $\int_0^1 \mathcal{E}(\alpha) d\alpha$. We also consider generalized error sum functions for the approximation with algebraic numbers of bounded degrees in the sense of K.Mahler.

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