ON THE VALUE DISTRIBUTION OF ERROR SUMS FOR APPROXIMATIONS WITH RATIONAL NUMBERS

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Let α be a real number with convergents p_m/q_m from the continued fraction expansion of α. In this paper we investigate the functions $\mathcal{E}(\alpha) := \sum_{m\geq 0} |\alpha q_m - p_m|$ and $\mathcal{E}^*(\alpha) := \sum_{m\geq 0}(\alpha q_m - p_m)$ depending only on α and prove that their values are dense in $[0, (1+\sqrt{5})/2]$ and $[0, 1]$, respectively. For any sequence $(\alpha_\mu)_{\mu\geq 1}$, which is uniformly distributed modulo 1, we show that both sequences $(\mathcal{E}(\alpha_\mu))_{\mu\geq 1}$ and $(\mathcal{E}^*(\alpha_\mu))_{\mu\geq 1}$ are not uniformly distributed. Among other things the proofs rely on an inequality for the function $\mathcal{E}(\alpha)$, which improves a former result of the first named author.

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