RESULTS AROUND RUBEL'S UNIVERSAL DIFFERENTIAL EQUATION

Carsten Elsner

New Directions in Value-Distribution Theory of Zeta and L-Functions, Conference Proceedings; Würzburg Conference, October 6-10, 2008; Rasa Steuding, Jörn Steuding (Eds.), Shaker-Verlag, Aachen 2009, 39-55.

In 1981, L.A.Rubel asked whether there exists a so-called algebraic universal differential equation such that every real continuous function f defined on the real line can be uniformly approximated with respect to the supremum norm by analytic solutions of this differential equation. This problem is still unsolved. Rubel himself found an algebraic differential equation whose $C^\infty(\mathbb{R})$-solutions can be used to approximate uniformly any continuous function f on the real line. In this paper we provide a survey on various aspects of contributions to the theory of universal differential and functional-differential equations. For instance, Rubel's conjecture can be proven under two restrictions: We consider continuous functions f satisfying $|f(x)| = \mathcal{O}(\exp((1-\delta)x)$ for $|x| \to \infty$, $\delta > 0$, and use a suitable integral norm instead of the supremum norm. Moreover, a former result of the author dealing with a universal partial differential equation is generalized for the approximation of meromorphic functions in several variables having poles of bounded order.

MR 2000 Subject Classification: 34A34, 34C11, 41A15, 41A63

Key words: Universal equations, algebraic differential equations, approximation of functions, analytic functions