# ON A GENERALIZATION OF A FORMULA OF SER AND APPLICATIONS TO THE RIEMANN ZETA FUNCTION AND TO DIRICHLET L-SERIES

Carsten Elsner

**(Analysis 30 (2010), 411–431; DOI 10.1524/anly.2010.1063)**

Let $s_n = 1 + 1/2 + \cdots + 1/(n-1) - \log n$. In 1995 and 2006, the author has found series transformations of the type $\sum_{k=0}^{n} \mu_{n,k,\tau_1} s_{k+\tau_2}$ with integer coefficients $\mu_{n,k,\tau_1}$, from which follows geometric convergence to Euler's constant $\gamma$ for $\tau_1, \tau_2 = \mathcal{O}(n)$. In recently published papers, *T.Rivoal* and *Kh.&T.Hessami Pilehrood* have generalized these results. One starting point for such investigations is the formula of *J.Ser*, which expresses the remainder $\gamma - s_n$ by a rational series. In this paper we generalize Ser's formula to Stieltjes constants of order zero and to the values of the Riemann zeta function and Dirichlet $L$-series at integer points $2, 3, \ldots$. We investigate a linear series transformation $\sum_{k=0}^{n} \mu_{n,k,\tau} \sum_{m=1}^{k+\tau} 1/m^r$ which converges quickly to $\zeta(r)$ for any fixed positive parameter $\tau$ when $n$ tends to infinity.

*MR 2000 Subject Classification:* 11J04, 26D15, 33F05, 33B15, 11M06