A REMARK ON NESTERENKO'S THEOREM FOR RAMANUJAN FUNCTIONS

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We study the algebraic independence of values of the Ramanujan q-series

$$A_{2j+1}(q) = \sum_{n=1}^{\infty} n^{2j+1} q^{2n}/(1-q^{2n})$$

or $S_{2j+1}(q)$ $(j \geq 0)$. It is proved that, for any distinct positive integers i, j satisfying $(i,j) \neq (1,3)$ and for any $q \in \overline{\mathbb{Q}}$ with $0 < |q| < 1$, the numbers $A_1(q)$, $A_{2i+1}(q)$, $A_{2j+1}(q)$ are algebraically independent. Furthermore, the q-series $A_{2i+1}(q)$ and $A_{2j+1}(q)$ are algebraically dependent over $\overline{\mathbb{Q}}(q)$ if and only if $(i,j) = (1,3)$.

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