

ON CONVERGENTS FORMED FROM DIOPHANTINE EQUATIONS

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We compute upper and lower bounds for the approximation of certain values ξ of hyperbolic functions by rationals x/y such that x, y satisfy Diophantine equations. We show that there are infinitely many coprime integers x, y such that

$$|y\xi - x| \ll \frac{\log \log y}{\log y}$$

and a Diophantine equation holds simultaneously for some integer z . Conversely, all positive integers x, y with $y \geq c_0$ solving the Diophantine equation satisfy

$$|y\xi - x| \gg \frac{\log \log y}{\log y}.$$

Moreover, we approximate $\sin(\pi\alpha)$ and $\cos(\pi\alpha)$ by rationals in connection with solutions of a quadratic diophantine equation when $\tan(\pi\alpha/2)$ is a Liouville number.

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