ON PRIME-DETECTING SEQUENCES FROM APÉRY'S RECURRENCE FORMULAE FOR $\zeta(3)$ and $\zeta(2)$

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We consider the linear three-term recurrence formula

$$X_n = (34(n-1)^3 + 51(n-1)^2 + 27(n-1) + 5)X_{n-1} - (n-1)^6 X_{n-2} \quad (n \geq 2)$$

corresponding to Apéry's non-regular continued fraction for $\zeta(3)$. It is shown that integer sequences $(X_n)_{n\geq 0}$ with $5X_0 \neq X_1$ satisfying the above relation are prime-detecting, i.e. $X_n \not\equiv 0 \,(\bmod\, n)$ if and only if n is a prime not dividing $|5X_0 - X_1|$. Similar results are given for integer sequences satisfying the recurrence formula

$$X_n = (11(x-1)^2 + 11(x-1) + 3)X_{n-1} + (n-1)^4 X_{n-2} \quad (n \geq 2)$$

corresponding to Apéry's non-regular continued fraction for $\zeta(2)$ and for sequences related to $\log 2$.

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