A RECURRENCE FORMULA FOR LEAPING CONVERGENTS OF NON-REGULAR CONTINUED FRACTIONS

Carsten Elsner and Takao Komatsu

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Given a continued fraction $[a_0; a_1, a_2, \ldots]$, $p_n/q_n = [a_0; a_1, a_2, \ldots, a_n]$ is called the n-th convergent for $n = 0, 1, 2, \ldots$. Leaping convergents are those of every r-th convergent p_{rn+i}/q_{rn+i} $(n = 0, 1, 2, \ldots)$ for fixed integers r and i with $r \geq 2$ and $i = 0, 1, 2, \ldots, r-1$. This leaping step r can be chosen as the length of period in the continued fraction. The first named author studied the leaping convergents p_{3n+1}/q_{3n+1} for the continued fraction of $e = [2; \overline{1, 2k, 1}]_{k=1}^{\infty}$ and obtained some arithmetical properties. The second named author studied those p_{3n}/q_{3n} for $e^{1/s} = [1; \overline{s(2k-1)-1, 1, 1}]_{k=1}^{\infty}$ $(s \geq 2)$. He has also extended such results for some more general continued fractions. Such concepts have been generalized in the regular continued fractions. In this paper leaping convergents in the non-regular continued fractions are considered so that a more general three term relation is satisfied. Moreover, the leaping step r need not necessarily to equal the length of period. As one of applications a new recurrence formula for leaping convergents of Apery's continued fraction of $\zeta(3)$ is shown.