ON RATIONAL APPROXIMATIONS TO EULER'S CONSTANT γ AND TO $\gamma + \log(a/b)$

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In this paper the author continues to study series transformations for the Euler - Mascheroni constant γ. Here, we discuss in detail recently published results of A.I.Aptekarev and T.Rivoal who found rational approximations to γ and $\gamma + \log q$ ($q \in \mathbb{Q}_{>0}$) defined by linear recurrence formulae. The main purpose of this paper is to adapt the concept of linear series transformations with integral coefficients such that rationals are given by explicit formulae which approximate γ and $\gamma + \log q$. It is shown that for every $q \in \mathbb{Q}_{>0}$ and every integer $d \geq 42$ there are infinitely many rationals a_m/b_m for $m = 1, 2, \ldots$ such that

$$\Big| \gamma + \log q - \frac{a_m}{b_m} \Big| \ll \left(\frac{(1 - 1/d)^d}{(d-1)4^d} \right)^m$$

and $b_m | Z_m$ with $\log Z_m \sim 12d^2m^2$ for m tending to infinity.

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