

ON SUMS WITH BINOMIAL COEFFICIENTS

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Abstract. We give a survey on recurrence formulae for sums of the type

$$\sum_{0 \leq k \leq n} \binom{n}{k}^a \binom{n+k}{k}^b$$

for integers $a, b \geq 0$. A correlation is given between the sum corresponding to $a = 2, b = 3$ and the sum

$$\sum_{0 \leq k \leq n} k \cdot \binom{n}{k}^2 \binom{n+k}{k}^3$$

by a five-term recurrence formula.

Furthermore, we consider infinite sums of the type

$$\sum_n \frac{q_n}{\binom{2n}{n}} \quad \text{and} \quad \sum_n \frac{\binom{2n}{n}}{v_n},$$

where $(q_n)_n$ and $(v_n)_n$ denote sequences of rational and integral numbers. We compute the limits of

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+3) \cdots (2n+2k+1) \binom{2n}{n}}$$

and of the corresponding alternating sum.