# ON A UNIVERSAL DIFFERENTIAL EQUATION FOR THE ANALYTIC TERMS OF $C^\infty$-SUPERPOSITIONS ON THE REAL LINE

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Let $\omega \in C(\mathbb{R}) \to \mathbb{R}_{>0}$ denote a bounded continuous weight function taking positive values everywhere, such that

$$\int_{-\infty}^{\infty} \omega(x)\, dx \;=\; 1$$

holds. Then, by

$$\|g\|_\omega \;:=\; \int_{-\infty}^{\infty} \omega(x)|g(x)|\, dx \qquad (g \in C(\mathbb{R}))\;,$$

a norm for continuous functions $g$ on the real line is given.

**Theorem 1** *There exists a nontrivial autonomous algebraic differential equation $P = 0$ of order at most 7, where $P$ denotes an effectively computable polynomial in at most eight variables, having the following property.*
*Let $f \in C(\mathbb{R}) \to \mathbb{R}$ be some continuous function defined on the real line and let $\varepsilon$ be some arbitrary positive number. Then a superposition $H \in C^\infty(\mathbb{R})$ of analytic functions $H_r \in C^\omega(\mathbb{R})$ exists, say*

$$H(x) \;=\; \sum_{-\infty<r<\infty} H_r(x) \qquad (x \in \mathbb{R})\;,$$

*such that $\|f - H\|_\omega < \varepsilon$ holds, and every analytic function $H_r$ solves the above universal differential equation. Moreover, every analytic function $H_r$ on $\mathbb{R}$ is an entire function on $\mathbb{C}$.*